Math 201, **Quiz II: NEW Double Sample** *K. Aziziheris & N. Nahlus (2013)*

1) Let $f(x,y) = \sqrt{9 - x^2 - y^2}$

b) From definitions, show that $f_x\ (0, 0) = 0 = f_y\ (0, 0)$

c) Prove that f is differentiable at (0, 0) d) sketch the graph of $z = \sqrt{9 - x^2 - y^2}$ (upper sphere)

a) Find the domain of f and describe the level curves of f.

1**) Let $g(x,y) = \sqrt{9 - x^2 - y^2}$ +5x +5y +1

i) From definitions, show that $f_x\ (0, 0) = 5 = f_y\ (0, 0)$

ii) Prove that f is differentiable at (0, 0)

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2) (i) Show that $\lim\limits_{(x,y\to(0,0)} \dfrac{xy^9}{x^2+y^8} \cos(\dfrac{y}{x}) = 0.$ (ii) Let $\mathrm{f}\,(\mathrm{x},\mathrm{y}) = 7\cos(\dfrac{\mathrm{x}^{4/3}\mathrm{y}^4}{\mathrm{x}^2+\mathrm{y}^8})$ for $(\mathrm{x},\mathrm{y}) \neq (0,0)$

<u>Prove or disprove</u> that $f(0,0)$ can be defined so that $f(x,y)$ is continous at (0, 0).

2**) *Let f(x,y)=0 if x =0 or y=0 and f(x,y)=1 otherwise.*

i) Investigate lim f(x,y) as (x,y) goes (0,0).

ii) Investigate lim g(x,y)= x^2/(|x|+|y|) as (x,y) goes (0,0).

3*) Suppose $F(x,y,z,w)=100$ and all components of ∇F are never zero.

Find $\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial y}\cdot\frac{\partial y}{\partial x}$ given that $\frac{\partial x}{\partial z}=e^{3x-10y+7z}$. Justify your answer .

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4) The function $f(x,y,z)$ at a point p INcreases most rapidly in the direction of the vector v=(3,4,5) with directional derivative $10\sqrt{2}$.
(i) Find $\nabla f(P)$ (ii) Find the directional derivative of $f(x,y,z)$ at P in the direction of the vector w=(4, 0, 3).
(iii) Is it possible to find a vector v such that D_v (f) (P) = 20 ? Explain.

4**) The function $f(x,y,z)$ at a point p DEcreases most rapidly in the direction of the vector v=(3,4,5) with directional derivative $-10\sqrt{2}$. (i) Find $\nabla f(P)$ (ii) Find the directional derivative of $f(x,y,z)$ at P in the direction of the vector w=(4, 0, 3).
(iii) Is it possible to find a vector v such that D_v (f) (P) = 15 ? (Hint: $\sqrt{2}$ = 1.4.14..). Explain.

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5) The derivative of of *f*(*x; y*) at *P*(1*;* 2) in the direction of i + j is $2\sqrt{2}$
and in direction of –2j is –3. Find the derivative of *f* in the direction of –i–2j.
(big Hint: Suppose grad(f)(P)= <a, b>. So you have 2 equations in 2 unknowns

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0) Find a, b if $f(x,y,z)=e^{ax+by}\cos 5z$ satisfies Laplace equation $f_{xx}+f_{yy}+f_{zz}=0$.

0**) Prove that $f(x,y,z)=(x^2+y^2+z^2)^{1/2}$ satisfies Laplace equation $f_{xx}+f_{yy}+f_{zz}=0$.

NEW Baby 6**) Given the surface $z - x^2 + 4xy = y^3 + 4y - 2$ containing the point $P(1; -1; -2)$

a) Find an equation of the tangent plane to the surface at P.

b) Find an equation of the normal line to the surface at P.

7a) Investigate the critical points of

$f(x, y) = 2x^3 + 6xy + 2y^3 + 17$ for local maxima, local minima, or saddle points.

7b) Locate all local extrema and saddle points of $f(x, y) = x^3 - y^3 - 2xy + 6$

7c) Locate all local extrema and saddle points of $f(x, y) = 4xy - x^4 - y^4$.

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8) Find the parametric equations for the line tangent to the curve of intersection of the surfaces $xyz = 1$ and $x^2 + y^2 + 3z^2 = 5$ at the point $P(1; 1; 1)$. (big Hint: use cross products).

9) By about how much will $f(x; y; z) = \ln\sqrt{x^2 + y^2 + z^2}$ change if the point $p(x; y; z)$ moves from $P_0(3; 4; 12)$ a distance of 0.1 units in the direction of $3i + 6j - 2k$? (see Thomas page 794)

9**) By about how much will $f(x; y; z) = \ln\sqrt{x^2 + y^2 + z^2}$ change if the point $p(x; y; z)$ moves from $P_0(3; 4; 12)$ to $P_1(3.01; 4.03; 12.01)$.
Big Hint: You may use the "sister" formula: $\Delta f \sim f_x(P_0)\,\Delta x + \cdots \ldots \Delta y + \cdots \ldots \Delta z$

10) Find the set of points <u>on the surface</u> $x^2 + y^2 - 36 = 8xyz$ where the tangent plane is

(i) perpendicular to the x-y plane. (ii) parallel to the x-y plane.

11) (14.8) Use **Lagrange multipliers** to find the absolute maximum and minimum of the function $f(x, y, z) = 5x - 2y + z + 17$ on the surface $x^2 + y^2 + z^2 = 30$

12) (Chain Rule) Suppose $\nabla f(1,1,1) = \mathbf{5i} + 3\mathbf{j} + 4\mathbf{k}$ and $f(1,1,1)=5$.

Let $P = f(t^4, t^2, tx^2)$ where $f(u,v,w)$ is a differentiable function. Then at t=1, x=1,

(i) $\partial P/\partial t = \ldots\ldots$ (ii) $\partial(x^3 P)/\partial t = \ldots.$ (iii) $\partial(t^3 P)/\partial t = \ldots\ldots.$

12**) Suppose $f(tx, ty) = t^5 f(x,y)$ for all values of x, y, t (where $f(u,v)$ is a differentiable function). Show that (i) $xf_x + yf_y = 5f$ (Hint: Partial w.r.t t both sides, then set t=1).

(ii) $x^2 f_{xx} + 2xyf_{xy} + y^2 f_{yy} = 20f$ (Hint: Double partial w.r.t t both sides, then set t=1).

12***) All lecture/recitation problems on 14.4 (next week) (check them all out)

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13) **POSTPONED for Final Exam.** **POSTPONED for Final Exam.**

Find the absolute minimum and maximum of the function $f(x,y) = x^2 + 2y^2 - y - 1$

a) Over the region $R = \{(x,y): \ x^2 + y^2 \le 1\}$

b) Over the region $R = \{(x,y): \ x^2 + y^2 \le 1 \text{ and } y \ge 0\}$

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14) Taylor's Remainder's formula:

Type 1: Estimate exp(0.2)

Type 2: Show that f(x)= its Mac series given that $|f^{(n)}(x)| \le (5000000)^n \ for\ all\ x$

Type 3: Estimate exp (- 0.2). (Here you can manage with AST estimates)

Good Luck